量子力学无基础入门小抄

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1 课程大纲概念地图

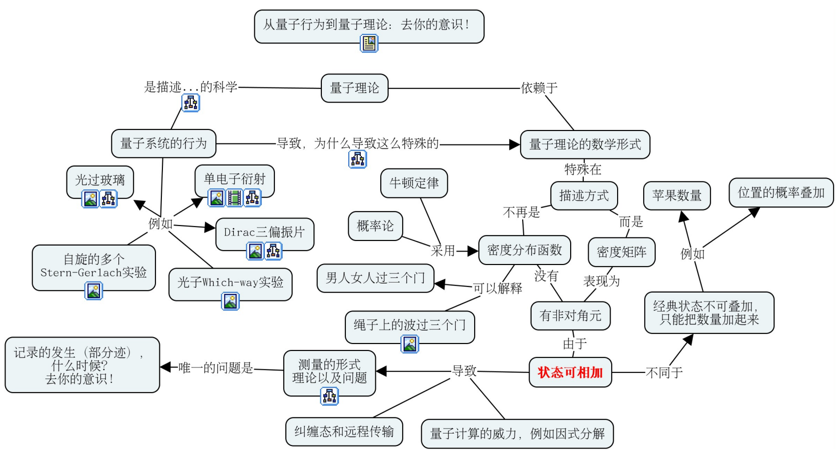

Figure 1: 课程内容概念地图

2 经典量子概率论数学框架对比

1. **经典状态**独立**没有加法**；**量子状态**不独立，**有加法**
2. 状态是密度矩阵ρ，相同，经典对角量子非对角由状态的差异造成
3. 物理量是算符A，相同，经典对角量子非对角，由状态的差异造成
4. 对某一状态ρ测量某一物理量A，均值遵循$\left\langle A\right\rangle_{\rho}=tr\left(A\rho\right)$，相同
5. 测量后状态相同：$\rho_{afm}=\left|m\right\rangle\left\langle m\right|\rho_{bfm}\left|m\right\rangle\left\langle m\right|\sim\left|m\right\rangle\left\langle m\right|$
6. 系统的演化相同：$\rho_{f}=A\rho_{i}A^{\dagger}$

除了量子状态有加法而经典状态不能相加天生就正交之外，两者没有任何不同！

3 经典随机状态数学模型基本框架

1. 可能的状态记为：$\{\alpha\}$，例如上下、正反、± 1、$1, 0$，满足
$$\langle\alpha|\ \beta\rangle = \delta_{\alpha\beta} \tag{1}$$
2. 状态的数学描述是密度分布函数：
$$\rho^c = \sum_\alpha p_\alpha \left|\alpha\right\rangle \left\langle\alpha\right| \tag{2}$$
3. 所需要测量的量是对角矩阵：
$$A = \sum_\alpha \alpha \left|\alpha\right\rangle \left\langle\alpha\right| \tag{3}$$
4. 在这个状态ρ^c下，测量A之后得到的均值为
$$\langle A\rangle_{\rho^c} = tr\left(A\rho^c\right) \tag{4}$$
5. 测量后状态就是所记录到的状态，也就是
$$\rho^c_{afm} = \left|m\right\rangle \left\langle m\right| \rho^c_{bfm} \left|m\right\rangle \left\langle m\right| \sim \left|m\right\rangle \left\langle m\right| \tag{5}$$
6. 状态ρ在操作A作用下的变化为
$$\rho^c_f = A\rho^c_i A^\dagger \tag{6}$$

4 量子概率论基本框架

1. 量子状态不独立，不正交，有加法
$$\langle\mu\ |\nu\rangle \neq \delta_{\mu\nu} \tag{7}$$
可以取任意正交归一基矢来展开，例如$\left|+\right\rangle = \frac{\sqrt{2}}{2}\left(\left|\uparrow\right\rangle + \left|\downarrow\right\rangle\right)$
2. 状态是密度矩阵ρ，可能有非对角元素，例如
$$\rho = \left|+\right\rangle \left\langle+\right| = \frac{1}{2}\left(\left|\uparrow\right\rangle\left\langle\uparrow\right| + \left|\uparrow\right\rangle\left\langle\downarrow\right| + \left|\downarrow\right\rangle\left\langle\uparrow\right| + \left|\downarrow\right\rangle\left\langle\downarrow\right|\right) \tag{8}$$
3. 物理量是算符A，可能有非对角元素，例如$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
4. 对某一状态ρ测量某一物理量A，均值遵循
$$\langle A\rangle_\rho = tr\left(A\rho\right) \tag{9}$$
5. 测量后状态：记录到哪一个可能状态，则系统在测量之后就处于相应的本征态，
$$\rho_{afm} = \left|m\right\rangle \left\langle m\right| \rho_{bfm} \left|m\right\rangle \left\langle m\right| \sim \left|m\right\rangle \left\langle m\right| \tag{10}$$
6. 系统的演化满足
$$\rho_f = A\rho_i A^\dagger = e^{-iHt}\rho_i e^{iHt} \tag{11}$$

其他关键词：以学科大图景为目标的以批判性思维系联性思考为基础的概念地图为技术的理解型学习、科学就是关于世界的可计算的可检验（可证伪）的模型、科学知识（模型和概念）一般还具有系统性、数学是思维的语言是描述世界的语言、学习是为了提出问题和面对问题为了创造所以要体验式创造式地来学习

5 基本运算：矢量相乘，矩阵相乘，部分迹

1. 最基本的运算：

$$|\mu\rangle|\nu\rangle = |\mu\nu\rangle, \tag{12}$$

$$\langle\mu|\,\nu\rangle = \delta_{\mu\nu}. \tag{13}$$

2. Dirac符号形式的矢量相乘：对于矢量，

$$|\psi\rangle = \begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix} = \psi_1|1\rangle + \psi_2|2\rangle, |\phi\rangle = \begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix} = \phi_1|1\rangle + \phi_2|2\rangle, \tag{14}$$

有（忽略复数的共轭，当做实数）

$$\langle\psi|\,\phi\rangle = \left(\sum_\mu \psi_\mu\langle\mu|\right)\left(\sum_\nu \phi_\nu|\nu\rangle\right) = \sum_{\mu\nu}\psi_\mu\phi_\nu\delta_{\mu\nu} = \sum_\mu \psi_\mu\phi_\mu, \tag{15}$$

$$|\psi\rangle|\phi\rangle = \left(\sum_\mu \psi_\mu|\mu\rangle\right)\left(\sum_\nu \phi_\nu|\nu\rangle\right) = \sum_{\mu\nu}\psi_\mu\phi_\nu|\mu\nu\rangle. \tag{16}$$

$$\langle\psi|\,\phi\rangle = \begin{bmatrix}\psi_1, \psi_2\end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix} = \psi_1\phi_1 + \psi_2\phi_2, \tag{17}$$

$$|\psi\rangle|\phi\rangle = \begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix} = \begin{bmatrix}\psi_1\phi_1\\ \psi_1\phi_2\\ \psi_2\phi_1\\ \psi_2\phi_2\end{bmatrix}. \tag{18}$$

3. Dirac符号形式的矩阵相乘：对于矩阵

$$A = \begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} = a_{11}|1\rangle\langle 1| + a_{12}|1\rangle\langle 2| + a_{21}|2\rangle\langle 1| + a_{22}|2\rangle\langle 2|, \tag{19}$$

$$\rho = \begin{bmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{bmatrix} = \rho_{11}|1\rangle\langle 1| + \rho_{12}|1\rangle\langle 2| + \rho_{21}|2\rangle\langle 1| + \rho_{22}|2\rangle\langle 2|, \tag{20}$$

有

$$A\rho = \begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{bmatrix}, \tag{21}$$

$$= \begin{bmatrix}a_{11}\rho_{11} + a_{12}\rho_{21} & a_{11}\rho_{12} + a_{12}\rho_{22}\\ a_{21}\rho_{11} + a_{22}\rho_{21} & a_{21}\rho_{12} + a_{22}\rho_{22}\end{bmatrix} \tag{22}$$

$$\begin{aligned} A\rho = & (a_{11}|1\rangle\langle 1| + a_{12}|1\rangle\langle 2| + a_{21}|2\rangle\langle 1| + a_{22}|2\rangle\langle 2|) \\ & \times(\rho_{11}|1\rangle\langle 1| + \rho_{12}|1\rangle\langle 2| + \rho_{21}|2\rangle\langle 1| + \rho_{22}|2\rangle\langle 2|), \end{aligned} \tag{23}$$

$$\begin{aligned} = & (a_{11}\rho_{11} + a_{12}\rho_{21})|1\rangle\langle 1| + (a_{11}\rho_{12} + a_{12}\rho_{22})|1\rangle\langle 2| \\ & + (a_{21}\rho_{11} + a_{22}\rho_{21})|2\rangle\langle 1| + (a_{21}\rho_{12} + a_{22}\rho_{22})|2\rangle\langle 2|. \end{aligned} \tag{24}$$

4. 求迹

$$tr\,(A\rho) = \sum_\mu \langle\mu|\,A\rho\,|\mu\rangle = a_{11}\rho_{11} + a_{12}\rho_{21} + a_{21}\rho_{12} + a_{22}\rho_{22}. \tag{25}$$